

A RANDOM REGRESSION COEFFICIENT TWO-FOLD FAY-HERRIOT MODEL FOR THE SMALL AREA ESTIMATION OF POVERTY INDICATORS

Naomi Diz-Rosales¹, María José Lombardía² and Domingo Morales³

¹Universidade da Coruña, CITIC, Spain.

²Universidade da Coruña, CITIC, Spain.

³Universidad Miguel Hernández de Elche, IUICIO, Spain.

ABSTRACT

Small Area Estimation refers to statistical methods designed to produce reliable estimates in domains where sample sizes are too small to yield precise direct estimates. Such domains, also called areas, can be geographical units (e.g., provinces, municipalities) or population subgroups (e.g., women aged 65 or over). To improve accuracy, these methods combine survey data with auxiliary information from administrative records, censuses, or other sources. A widely used tool in this field is the Fay-Herriot model, an area-level model that links direct survey estimates with auxiliary covariates while incorporating random effects to account for unexplained heterogeneity across areas. This approach produces empirical best linear unbiased predictors, which reduce variance compared to direct estimates and provide more stable results. In this work, we introduce an extended two-fold (doubly nested) Fay-Herriot model that allows for both random intercepts and random regression coefficients, thereby capturing heterogeneity not only in baseline levels across areas but also in the relationships between covariates and the target indicators. Based on this specification, we derive predictors of poverty indicators along with analytic and bootstrap-based estimators of mean squared error, both with and without bias correction. Model parameters are estimated via residual maximum likelihood, and the empirical best linear unbiased predictors are obtained for the random components. Simulation experiments are conducted to evaluate the behavior of the estimation procedure, the predictors, and the proposed uncertainty measures. The methodology is implemented using data from the 2022 Spanish Living Conditions Survey to obtain provincial estimates of poverty rates, disaggregated by sex and age group. In addition, we extend the model by incorporating temporal dependence at the intercept level, which allows the analysis to capture dynamic patterns across years. This temporal extension is applied to the estimation of average income in Spain by province and sex over the period 2013-2022. The proposed approach provides a flexible and robust tool for poverty and income mapping, offering accurate estimates together with reliable measures of precision.

Keywords: Bootstrap, Poverty Indicators, Random regression coefficient mixed models, Small Area Estimation.

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